Fundamental Principle of Counting/Multiplication rule

Q1) How many numbers greater than 999 but not greater than 5000 be formed using the digits 0,1,2,3,4 and 5 if (a) repetition of digits is allowed (b) repetition of digits is not allowed.

Q2) How many 5 digit numbers divisible by 5 can be formed using the digits 0, 1, 2,3,4,5 and 6 if (a) repetition of digits is allowed (b) repetition of digits is not allowed.

Q3) An intelligence agency decides to form 2 digit codes with distinct digits. The code which is hand written on paper can however create confusion when read upside down. For example the code 91 may also appear 19 when read upside down. How many codes can be formed in which there is no such confusion?

Q4) There is an assignment involving 6 tasks and 6 people. Task 1 cannot be given to person 1 or 2. Task 2 has to be given to either person 3 or 4. In how many ways can the assignment be done if each person is to be assigned only 1 task?

Q) A man forgot the seven digit phone number of his friend but remembers the following: the first 3 digits from the left are either 242 or 472, the digit 7 comes only once and the number is an even number. If the man was to use trial and error method, what is the minimum number of numbers he has to dial before finally getting the correct number?

Ans. 1215 + 3645 = 4860

Q5) There are n families F_1, F_2....., Fn in the neighbourhood of X. The number of members in family F_n = n+1. X decides to invite at most one member from each family. If the total number of ways in which he can invite a total of one or more members from his neighbourhood is 2519, find the value of n?

Ans. n=5

Q6) How many 3 digit numbers are such that if one digit id 6, the immediate next is 7?

Ans. 8 + 7 + 448 = 465

Q) If all integers from 100 and 800 are written, how many times will the digit 2 come?

Ans. 20*6 + 39 = 159

Permutations and Combinations

Q1) There are 15 points in a plane of which 7 are collinear and the remaining 8 are non collinear. How many different triangles, circles and straight lines can be formed?

Q2) In how many ways is it possible to arrange all the letters of the word EQUATION such that (a) no two consonants are together (b) all the consonants are always together?

Q3) In how many ways is it possible to organize a mixed doubles tennis tournament involving six couples such that no husband and wife play in the same game?

Q4) In how many ways can a person invite 8 friends to a party if he has to invite (a) at least 1 of them (b) at least 2 of them?

Q5) Let S be the set of first 10 whole numbers. How many subsets of S can be formed such that each one of them contains 7 but none of them contains 8?

Q6) Let X =(1,2,3,4,5,6,7) and A=(3,4,5). If A ∧ B = (3, 4), how many subsets of X can B represent?

Ans. 2^4 =16, since B cannot have 5 in it.

Chess Board

Q1) How many squares and rectangles are there in an n x n and an m x n grid?

Q2) In how many ways is it possible to select two white squares from a chess board such that (a) they are always together in the same row or column (b) they are never together in the same row or column?

Q3) In how many ways is it possible to select one black and one white squares from a chess board such that (a) they are always together in the same row or column (b) they are never together in the same row or column.

Q4) In how many ways is it possible to select two unit squares from a chess board such that (a) they share a common edge (b) they share a common corner

Q5) In how many ways is it possible to select two unit squares from a chess board such that lie on the same diagonal?

Q6) In how many ways is it possible to place two rooks on a chessboard such that they do not attack each other?

Q7) In how many ways is it possible to two select three unit squares from a chess board such that only one of them shares an edge with the other two?

Q8) There is a 5 x 5 square grid, with each cell having a distinct identity. In how many ways can 3 identical coins be placed in the grid (with at most 1 coin in each cell) such that no 2 coins are in the same row or column?

Answer. (25 x 16 x 9)/3! = 600

Permutations of alike things

Q1) Let $10^5 < K < 10^6$, where K is an integer. Then how many different K exist such that the sum of the digits of K is (a) 3 (b) 4.

Q2) In how many ways can a person reach from A to B if only forward or upward movement is allowed?

				B
A				

Q) Consider a rectangular grid of lines formed by "m" parallel lines intersecting "n" parallel lines running perpendicular to them. How many different paths along the grid are possible from top right corner to the bottom left corner, if only downward and leftward motion is allowed?

Ans. [(m + n-2)!]/ (m-1)! X (n-1)!

Q) In how many ways can you go from A to B such that a certain portion of the grid should not be visited?

Q3) In how many different ways can four letters of the word EFFERVESCENCE be (a) selected (b) arranged?

Q4) In how many ways is it possible to place three identical coins in a 5 x 5 grid (with at most one coin in each cell), such that no two coins are in the same row or column?

Ans. 300 = (25x16x9)/3!

Rank of a word

Q1) Find the rank of the words (a) HEAT (b) WHEAT (C) FLOWER (d) MAYANK

Ans. (d) = 200

Circular permutations

Q1) In how many different ways can 5 men and 3 women be seated around a circular table such that no two women sit together?

Q2) Ten friends go out to a restaurant for dinner. There they see two circular tables -one with 6 chairs and the other with 4 chairs. In how many ways can they sit around the two tables?

Partition concept

Q1) How many positive and non negative integer solutions exist for: $10 < (a + b + c + d) < 20$

Q2) Find the number of terms in the binomial expansion of $(a + b + c + d)^{20}$.

Q3) In how many ways can 25 marbles be distributed among P, Q, R and S such that P gets at least one marble, Q gets at least 2 marbles, R gets at least 3 marbles and S gets at least four marbles?

Q4) How many numbers less than 10^5 are such that the sum of their digits is 10?

Q5) In how many ways can a thief steel 10 houses in a line such that no two of them are together/adjacent?

Q6) Ten soldiers are standing in a single row. Their commanding officer wants to select any three of them such that there are at least two soldiers between any two of the soldiers selected. In how many ways can this be done?

Ans. ${}^{6}C_{3} = 20$ since a + b + c + d = 7 where a, b, c, d are the gaps between the three soldiers selected

Miscellaneous

Q1) Five horses are competing in a race. How many different finishes are possible if ties are also allowed?

Q2) Derangements → In how many ways can 10 letters be posted in 10 envelopes such that only 4 are placed in the right envelopes.